हमारा विश्वास...
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JEE ADVANCED
September 2020

ANSWER KEY

MATHEMATICS _ PAPER - 1

MOTION™
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SECTION 1 (Maximum Marks : 18)

- This section contains **SIX** (06) questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :

Full marks	: +3	If ONLY the correct option is chosen;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: –1	In all other cases.

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1. Suppose a,b denote the distinct real roots of the quadratic polynomial $x^2 + 20x - 2020$ and suppose c,d denote the distinct complex roots of the quadratic polynomial $x^2 - 20x + 2020$. Then the value of $ac(a-c) + ad(a-d) + bc(b-c) + bd(b-d)$ is

(A) 0 (B) 8000 (C) 8080 (D) 16000

Ans. **D**

2. If the function $f:R\to R$ is defined by $f(x) = |x|(x-\sin x)$, then which of the following statements is TRUE ?

(A) f is one-one, but NOT onto (B) f is onto, but NOT one-one

(C) f is BOTH one-one and onto (D) f is NEITHER one-one NOR onto

Ans. **C**

3. Let the functions $f:R\to R$ and $g:R\to R$ be defined by

$f(x) = e^{x-1} - e^{-|x-1|}$ and $g(x) = \frac{1}{2}(e^{x-1}+e^{1-x})$.

Then the area of the region in the first quadrant bounded by the curves $y = f(x)$, $y = g(x)$ and $x = 0$ is.

(A) $(2-\sqrt{3})+\frac{1}{2}(e-e^{-1})$ (B) $(2+\sqrt{3})+\frac{1}{2}(e-e^{-1})$

(C) $(2-\sqrt{3})+\frac{1}{2}(e+e^{-1})$ (D) $(2+\sqrt{3})+\frac{1}{2}(e+e^{-1})$

Ans. **A**

4. Let a, b and λ be positive real numbers. Suppose P is an end point of the latus rectum of the parabola $y^2 = 4\lambda x$, and suppose the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through the point P. If the tangents to the parabola and the ellipse at the point P are perpendicular to each other, then the eccentricity of the ellipse is.

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{2}{5}$

Ans. **A**

5. Let C_1 and C_2 be two biased coins such that the probabilities of getting head in a single toss are $\frac{2}{3}$ and $\frac{1}{3}$, respectively. Suppose α is the number of heads that appear when C_1 is tossed twice, independently, and suppose β is the number of heads that appear when C_2 is tossed twice, independently. Then the probability that the roots of the quadratic polynomial $x^2 - \alpha x + \beta$ are real and equal, is

(A) $\frac{40}{81}$ (B) $\frac{20}{81}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Ans. **B**

6. Consider all rectangles lying in the region $\left\{(x, y) \in R \times R : 0 \le x \le \frac{\pi}{2} \text{ and } 0 \le y \le 2\sin(2x)\right\}$ and having one side on the x-axis. The area of the rectangle which has the maximum perimeter among all such rectangles, is

(A) $\frac{3\pi}{2}$ (B) π (C) $\frac{\pi}{2\sqrt{3}}$ (D) $\frac{\pi\sqrt{3}}{2}$

Ans. **C**

SECTION 2 (Maximum Marks : 24)

- This section contains **SIX** (06) questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four options(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correc answer(s).
- Answer to each question will be evaluated according to the following marking scheme :

Full marks	: +4	If only (all) the correct option(s) is (are) chosen;
Partial Marks	: +3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	: +2	If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: –2	In all other cases.

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7. Let the function $f : R \to R$ be defined by $f(x) = x^3 - x^2 + (x - 1)\sin x$ and let $g : R \to R$ be an arbitrary function. Let $fg : R \to R$ be the product function defined by $(fg)(x) = f(x)g(x)$. Then which of the following statements is/are TRUE ?

(A) If g is continuous at x = 1, then fg is differentiable x = 1
(B) If fg is differentiable at x = 1, then g is continuous at x = 1
(C) If g is differentiable at x = 1, then fg is differentiable at x = 1
(D) If fg is differentiable at x =1, then g is differentiable at x = 1

Ans. **A,C**

8. Let M be a 3 × 3 invertible matrix with real entries and let I denote the 3 × 3 identity matrix. If M^{-1} = adj (adj M), then which of the following statements is/are ALWAYS TRUE ?

(A) M = I (B) det M = 1 (C) $M^2 = I$ (D) (adj M^2) = I

Ans. **B,C,D**

9. Let S be the set of all complex numbers z satisfying $|z^2+z+1| = 1$. Then which of the following statements is/are TRUE ?

(A) $\left|z+\frac{1}{2}\right| \le \frac{1}{2}$ for all $z \in S$ (B) $|z| \le 2$ for all $z \in S$

(C) $\left|z+\frac{1}{2}\right| \ge \frac{1}{2}$ for all $z \in S$ (D) The set S has exactly four elements

Ans. **B,C**

10. Let x, y and z be positive real numbers. Suppose x, y and z are the lengths of the sides of a triangle opposite to its angles X, Y and Z, respectively. If $\tan\frac{X}{2} + \tan\frac{Z}{2} = \frac{2y}{x+y+z}$, then which of the following statements is/are TRUE ?

(A) 2Y = X + Z (B) Y = X + Z (C) $\tan\frac{x}{2} = \frac{x}{y+z}$ (D) $x^2 + z^2 - y^2 = xz$

Ans. **B,C**

11 Let L_1 and L_2 be the following straight lines.

$$L_1 : \frac{x-1}{1} = \frac{y}{-1} = \frac{z-1}{3} \text{ and } L_2 : \frac{x-1}{-3} = \frac{y}{-1} = \frac{z-1}{1}$$

Suppose the straight line $L : \frac{x-\alpha}{l} = \frac{y-1}{m} = \frac{z-\gamma}{-2}$

lies in the plane containing L_1 and L_2, and passes through the point of intersection of L_1 and L_2. If the line L bisects the acute angle between the lines L_1 and L_2, then which of the following statements is/are TRUE?

(A) $\alpha - \gamma = 3$ (B) $l + m = 2$ (C) $\alpha - \gamma = 1$ (D) $l + m = 0$

Ans. **A,B**

12. Which of the following inequalities is/are TRUE?

(A) $\int_0^1 x\cos x\,dx \geq \frac{3}{8}$ (B) $\int_0^1 x\sin x\,dx \geq \frac{3}{10}$

(C) $\int_0^1 x^2\cos x\,dx \geq \frac{1}{2}$ (D) $\int_0^1 x^2\sin x\,dx \geq \frac{2}{9}$

Ans. **A,B,D**

SECTION 3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/round -off the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme :

Full marks	: +4	If ONLY the correct numerical value is entered;
Zero Marks	: 0	In all other cases.

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13. Let m be the minimum possible value of $\log_3\left(3^{y_1}+3^{y_2}+3^{y_3}\right)$, where y_1, y_2, y_3 are real numbers for which $y_1+y_2+y_3=9$. Let M be the maximum possible value of $\left(\log_3 x_1+\log_3 x_2+\log_3 x_3\right)$, where x_1, x_2, x_3 are positive real numbers for which $x_1+x_2+x_3=9$. Then the value of $\log_2\left(m^3\right)+\log_3\left(M^2\right)$ is____

Ans. **8.00**

14. Let $a_1, a_2, a_3, \ldots$ be a sequence of positive integers in arithmetic progression with common difference 2. Also, let $b_1, b_2, b_3, \ldots$ be a sequence of positive integers in geometric progression with common ratio 2. If $a_1=b_1=c$, then the number of all possible values of c, for which the equality $2\left(a_1+a_2+\cdots+a_n\right)=b_1+b_2+\cdots+b_n$ holds for some positive integer n, is____

Ans. **1.00**

15. Let $f:[0,2]\to\mathbb{R}$ be the function defined by

$$f(x)=(3-\sin(2\pi x))\sin\left(\pi x-\frac{\pi}{4}\right)-\sin\left(3\pi x+\frac{\pi}{4}\right)$$

If $\alpha,\beta\in[0,2]$ are such that $\{x\in[0,2]: f(x)\geq 0\}=[\alpha,\beta]$, then the value of $\beta-\alpha$ is____

Ans. **1.00**

16. In a triangle PQR, let $\vec{a}=\overrightarrow{QR}, \vec{b}=\overrightarrow{RP}$ and $\vec{c}=\overrightarrow{PQ}$. If $|\vec{a}|=3,\ \ |\vec{b}|=4$ and $\dfrac{\vec{a}\cdot(\vec{c}-\vec{b})}{\vec{c}\cdot(\vec{a}-\vec{b})}=\dfrac{|\vec{a}|}{|\vec{a}|+|\vec{b}|}$,

then the value of $|\vec{a}\times\vec{b}|^2$ is____

Ans. **108.00**

17. For a polynomial $g(x)$ with real coefficients, let m_g denote the number of distinct real roots of $g(x)$. Suppose S is the set of polynomials with real coefficients defined by

$$S=\left\{\left(x^2-1\right)^2\left(a_0+a_1x+a_2x^2+a_3x^3\right): a_0,a_1,a_2,a_3\in\mathbb{R}\right\}$$

For a polynomial f, let f' and f'' denote its first and second order derivatives, respectively. Then the minimum possible value of $\left(m_{f'}+m_{f''}\right)$, where $f\in S$, is____

Ans. **5.00**

18. Let e denote the base of the natural logarithm. The value of the real number a for which the right hand limit $\lim\limits_{x\to 0^+}\dfrac{(1-x)^{\frac{1}{x}}-e^{-1}}{x^a}$ is equal to a nonzero real number, is____

Ans. **1.00**

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